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TASHKENT UNIVERSITY OF
INFORMATION TECHNOLOGIES
NAMED AFTER MUHAMMAD AL-KHWARIZMI

MUHAMMAD AL-XORAZMIY NOMIDAGI
TOSHKENT AXBOROT TEXNOLOGIYALARI
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Normakhmad Ravshanov, Abdugani Nematov, Shikhnazar Ismailov, Mohiniso Mahmudova
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Algorithm and program for solving the gas filtration problem in dynamically interconnected porous media

Normakhmad Ravshanov
Department of Multimedia Technologies
Tashkent university of information technologies named after Muhammad al-Khwarizmi
Tashkent city 100 000, Uzbekistan
Email:

Abdugani Nematov
Department of Multimedia Technologies
Tashkent university of information technologies named after Muhammad al-Khwarizmi
Tashkent city 100 000, Uzbekistan
Email: abdun@rambler.ru

Shikhnazar Ismailov
Department of Multimedia Technologies
Tashkent university of information technologies named after Muhammad al-Khwarizmi
Tashkent city 100 000, Uzbekistan
*Email: *shixnazar.ismailov@gmail.com*

Mohiniso Mahmudova
Department of Multimedia Technologies
Tashkent university of information technologies named after Muhammad al-Khwarizmi
Tashkent city 100 000, Uzbekistan
Email: mahmudova_mohiniso@mail.ru

Abstract. This article examines the mathematical modeling of the thermal impact process on an oil reservoir when water vapor is pumped into it. The problem of heat propagation in a porous medium is being investigated, taking into account heat losses through the roof and bottom of the layer, which leads to a more realistic description of temperature fields. The model is based on a system of differential equations with phase transitions and takes into account the thermal anisotropy of rocks. Numerical methods using finite-difference schemes and the distillation method were used to solve the Stefan problem. A computational experiment was conducted, the results of which showed a significant influence of heat loss on the temperature field and the efficiency of oil displacement. The obtained data were compared with previously developed models, key differences were identified, and refined calculation approaches were proposed. The work has applied significance for increasing the oil recovery of highly viscous oil reservoirs.

Key words: *thermal effect, water vapor, oil reservoir, modeling, heat losses, Stefan's problem, numerical methods, phase transition, thermal conductivity, high-viscosity oil, finite differences.*

I. INTRODUCTION

As we know, the structure of global hydrocarbon reserves is viscous oil, which accounts for approximately 60% and, as low-viscosity oil deposits are developed, their extraction share increases over time. Addressing the problem of increasing oil recovery from

deposits with viscous oils is a pressing issue in hydrocarbon production worldwide.

One of the effective methods of impacting the reservoir to increase oil recovery from highly viscous oil deposits is a sufficiently tested method for shallow depths worldwide, which is the process of thermal impact on the reservoir by injecting water vapor into it. Works [1, 2, 3, 4, 5, 6, 7] are devoted to the theoretical and experimental study of heat distribution in a reservoir when water vapor is pumped into it.

As shown by the laboratory experiments conducted by Stovall on the models of the porous medium of petroleum sandstone [1], water vapor on the saturation line has a practically perfect displacement capacity.

One of the main advantages of water vapor is that the remaining oil, not displaced by the hot water formed as a result of vapor condensation,

evaporates under the influence of water vapor. Thus, additional moving energy is created [1,2,7,8].

The article [9] presents the results of a laboratory study of the thermal effect on the carbonate oil reservoir, examines the influence of the composition of the vapor-gas eluent, temperature, and pressure on the permeability of the carbonate rock, the composition of gaseous products of thermolysis and extracted oil, and the efficiency of its extraction from the carbonate rock.

In the work [10], comprehensive studies were conducted to increase oil recovery, and options for impacting the reservoir through a single horizontal well were considered.

Based on the research results, the following conclusions can be drawn: drilling a horizontal well for surface steam-thermal impact in the current conditions of the permocarbon deposit, especially in the heat-affected zone, is associated with a significant economic risk, which can be mitigated by cyclic steam injection technology; drilling a horizontal well in zones of high waterlogging and practically no oil in the well product before steam treatment of its bottomhole zone will allow increasing the flow rate of horizontal wells several times; drilling a horizontal well in the high-temperature zone is not recommended. The conducted research has shown the need for a thorough approach to selecting areas for laying horizontal wells. In addition to geophysical and hydrodynamic studies of drilled wells of the selected section for horizontal wells, detailed geological and hydrodynamic modeling of the processes occurring on it and selection of optimal technological parameters for each specific section are recommended, due to the difference in current geological and hydrodynamic conditions and technogenic factors (presence of injection, extraction, opening of objects, condition of the production column, cement stone, etc.).

The article [11] describes the process of mathematical modeling of the pyrophoric layer heating - one of the causes of explosions and fires in the oil and gas industry. A system of equations modeling the one-dimensional heating process of the pyrophoric layer is presented, using the heat conductivity equation as the main equation. Cooling conditions for the tank, both inside and outside, have been added. The calculated data of the mathematical model for the heating of the pyrophoric layer over time after reaching the maximum calculated self-heating temperature for various thicknesses of the pyrophoric deposits are presented and compared with experimental data. The temperature distribution in the layer of pyroforic deposits is shown.

In the work [12-13], a mathematical model of the technology of thermal impact on the oil reservoir in combination with acid treatment of the wellbore zone of the reservoir was proposed. A comparison of the oil recovery coefficient was carried out in the cases of a conventional facility with heat transfer, an acid-based facility, and a combined technology. It has been shown that the combined use of heat treatment and acid treatment is the most effective oil production regime.

As noted in the work [14] due to the depletion of currently actively developed oil fields, oil and gas companies are increasingly focusing on developing methods for developing heavy oil and natural bitumen deposits. The purpose of the work is to numerically model the heating of a high-viscosity oil reservoir by electric current to assess the effectiveness of electrical thermal impact as a method for intensifying oil production [14]. Electrothermal treatment of wells in heavy oil fields is carried out by supplying electric current directly to the bottomhole zone, while cable or electrically insulated pipe is used as a conductor supplying current to the oil-bearing reservoir. To identify the main patterns of electrical and thermal impact on the reservoir, a single vertical well and a system of four vertical wells, one of which is injection and three extraction, were considered. To reduce parasitic currents outside the productive collector, a special casing column design was used in the experiment, where the perforation area is separated from the rest of the column by electrically non-conductive inserts, or the outer surface of the column passing outside the productive layer is coated with an electrically insulating material.

The dissertation work [15] presents research in the field of studying complex filtration processes in fractured rocks using effective methods of modern mathematical physics and, on this basis, developing methods for solving practical problems of developing oil fields with carbonate reservoirs. The main issues of the research are reduced to studying the features of the movement of Newtonian and anomalous oils, heterogeneous immiscible fluids in purely fractured and fractured-porous reservoirs (near and outside the wells), substantiating and selecting a mechanical filtration model, developing methods for calculating the filtration flow in a system of arbitrarily located wells with unequal yields, solving, on this basis, initial, boundary, and initial-boundary problems, deriving formulas for determining the technological indicators of oil reservoir development with carbonate reservoirs, and using them in field exploitation, substantiating specific recommendations, and implementing the obtained results in industry.

Models of non-isothermal displacement of oil by an active impurity solution are considered, taking into account and without considering heat exchange with surrounding rocks. For the numerical solution of the two-phase three-component filtration equation, explicit difference schemes were used [16].

The method of decomposition by physical processes is used.

In the dissertation work [17], a simplified model of the cyclic steam-thermal impact process on the reservoir was developed, based on the integral equations of the heat balance in the reservoir and surrounding rocks, and it was established that varying the main times (stages) of the cyclic steam-thermal impact process can achieve maximum oil production intensification. The work also developed a methodology for optimizing the process based on the ratio of steam injection time to the production period. Optimization studies were conducted for the main development objects of the Stepnoozerskoye deposit. The dissertation author proposed a method for the numerical study of the periodic thermal polymer impact process, which allows for the study of the dynamics of oil displacement and assesses the degree of instability of displacement fronts. A methodology for predicting periodic thermal polymer exposure has been proposed, taking into account the influence of the main technological parameters (the number and size of edges, the order of fluid injection into the reservoir, etc.) on the process efficiency.

The article [18] is devoted to the mathematical modeling of the process of heating a carbonate-containing oil reservoir under the combined influence of hydrochloric acid and a high-frequency electromagnetic field. Calculations of the spatiotemporal distribution of temperature were performed, and an analysis of the contribution of various physical processes was carried out.

In the dissertation work of S.A. Kalinin [19], the methodology was formed, methods were developed, and the techniques and technology for experimental study of combined methods of extracting superviscous oil (SVA) based on thermal and gas exposure to the reservoir were improved, and the possibility of increasing the efficiency of developing a fractured-porous-cavernous reservoir with low reservoir temperature, saturated with SVA, under combined exposure to the reservoir with a heat carrier and carbon dioxide, by involving the matrix zone of the reservoir in more active development was scientifically substantiated. Also, based on the results of laboratory and numerical experiments on linear reservoir models, a methodology for determining the optimal conditions for implementing combined impact on SVN deposits has been developed. Optimal conditions have been established under which the greatest effectiveness of implementing the combined effect of the heat carrier and carbon dioxide in the conditions of the Permo-Carbon deposit of the Usin deposit is achieved.

In the work [20], a mathematical model of the technology of thermal impact on the oil reservoir in combination with acid treatment of the wellhead zone of the reservoir was developed, and a comparison of the oil recovery coefficient was carried out in cases of conventional flooding, flooding with the injection of a heat carrier, flooding

with the application of acid, and combined technology. It has been shown that the combined use of heat treatment and acid treatment is the most effective oil production regime.

As practice has shown, despite the diversity of known thermal methods for extracting superviscous oil, the basis of most of them is the process of displacing oil by steam or a mixture of hot water and steam, and it is precisely this circumstance that determines the constant attention paid to the process of steam-thermal impact on oil reservoirs. As noted by specialists in the field of oil reservoir design development, the main difficulties encountered in its theoretical study are related to the calculation of three-phase flows of water, oil, and steam, and the consideration of heat exchange between the reservoir and surrounding rocks. Until recently, these difficulties were insurmountable, and there was no detailed understanding of the structure of the oil displacement zone by steam under conditions of interconnected heat and mass transfer processes accompanied by phase transitions. The main approach to analyzing the processes of oil displacement by steam and numerical modeling of such problems is associated with the names of Coats K.N. (1974), Boxerman A.A. (1975), Zazovsky A.F. (1986), Rubinstein L.I. (1972), Yortsos Y.C. (1981), Marx J.W. (1959).

With the advent of powerful computing resources, opportunities have opened up for modeling the process of anomalous oil filtration, taking into account the displacement of high-viscosity oil by steam or a mixture of steam and hot water. Despite the above, modeling and solving these problems is a complex scientific problem, it is sufficient to indicate that the theory of non-isothermal three-phase filtration, taking into account phase transitions, is still far from being completed, and there are no proven programs for calculating these processes yet.

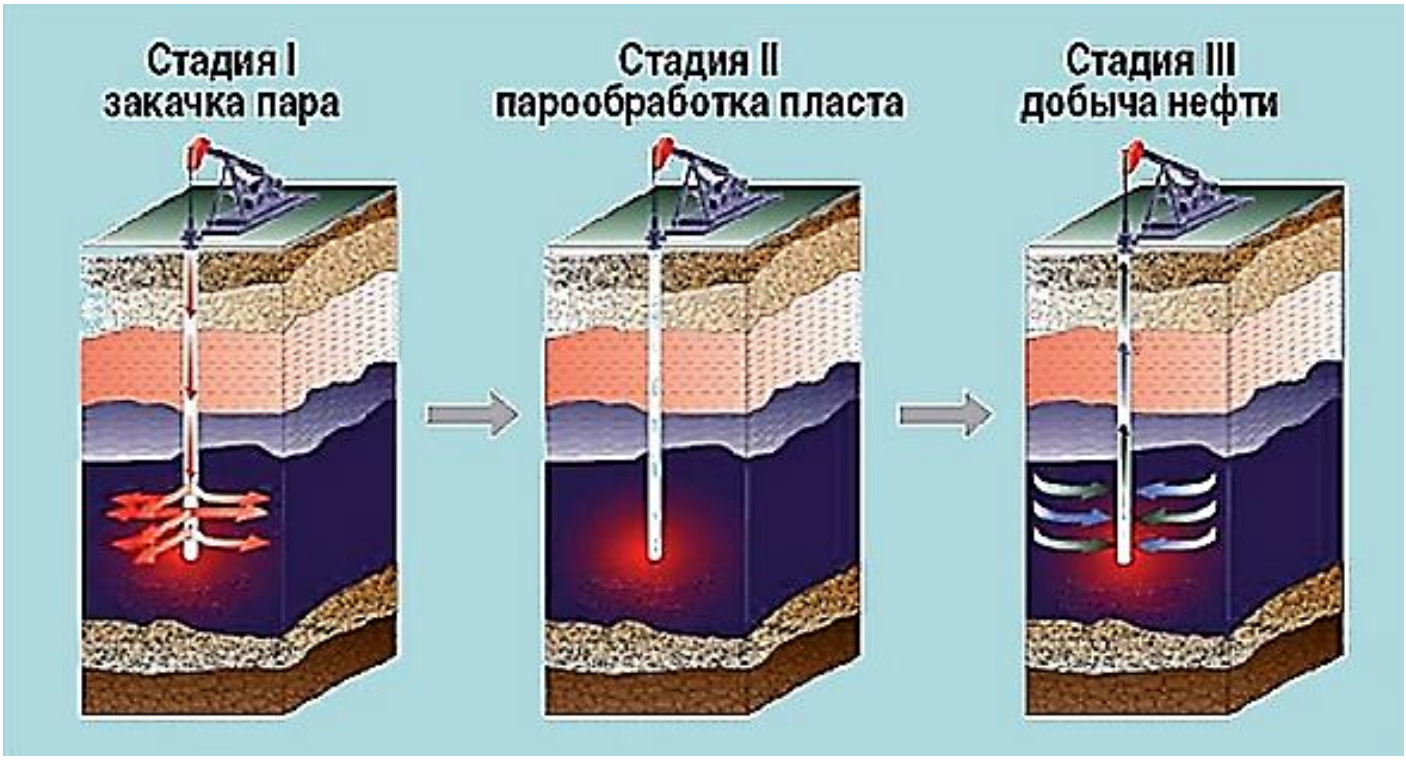


Fig. 1. Stages of thermal impact on the oil reservoir using steam injection.

When oil is displaced by steam in a porous medium, as a rule, three zones of temperature fields are formed: steam, condensate, and oil. Further, we will assume that the physical properties of the oil and condensation zones are the same. The rocks surrounding the layer are considered thermally anisotropic, i.e., the thermal conductivity of the rocks covering and underlying the layer in the horizontal direction is negligibly small, and the layer is anisotropic, with infinitely high thermal conductivity in the vertical direction.

Problem statement. Then, the solution of the problem of heat propagation in a layer during plane-radial filtration in a porous medium without considering the Joule-Thomson effect, the elasticity of the considered media from temperatures, can be reduced to solving a system of differential equations: [2, 4, 6]:

$$\lambda_2 \frac{\partial^2 u^1}{\partial z^2} = C_1 \frac{\partial u}{\partial t};$$

$$D_1 = \{m_2 \le z \le m, \quad 0 \le t \le T_{bp}\}; \tag{1}$$

$$\lambda_1^* \left(\frac{\partial^2 w^{(1)}}{\partial r^2} + \frac{1}{r} \frac{\partial w^{(1)}}{\partial r} \right) + \frac{\lambda_1}{H} \frac{\partial u^{(1)}}{\partial z} \Big|_{z=m_2} =$$

$$= \frac{\lambda_2}{H} \frac{\partial u^{(2)}}{\partial z} \Big|_{z=m_1} = C_1^* \left(\frac{\partial w^{(1)}}{\partial t} + v_1 \frac{\partial w^{(1)}}{\partial r} \right); \tag{2}$$

$$D_2 = \{r_0 \le r \le R(t), \quad 0 \le t \le T_{\beta p}\};$$

$$\lambda_2^* \left(\frac{\partial^2 w^{(1)}}{\partial r^2} + \frac{1}{r} \frac{\partial w^{(2)}}{\partial r} \right) + \frac{\lambda_1}{H} \frac{\partial u^{(1)}}{\partial z} \Big|_{z=m_2} -$$

$$- \frac{\lambda_2}{H} \frac{\partial u^{(2)}}{\partial z} \Big|_{z=m_1} = C_2^* \left(\frac{\partial w^{(2)}}{\partial t} + v_2 \frac{\partial w^{(2)}}{\partial r} \right); \tag{3}$$

$$D_2^* = \{R(t) \le r \le l, \quad 0 \le t \le T_{bp}\};$$

$$\lambda_2 \frac{\partial^2 u^{(2)}}{\partial z^2} = C_2 \frac{\partial u^{(2)}}{\partial t}; \tag{4}$$

$$D_3' = \{0 \le z \le m, \quad 0 \le t \le T_{bp}\};$$

under the following initial and boundary conditions:

$$u^{(1)}(r, z, 0) = f_1 \tag{5}$$

$$u^{(2)}(r, z, 0) = f_2 \tag{6}$$

$$u^{(1)}(r, m, t) = f_1 \tag{7}$$

$$u^{(2)}(r, 0, t) = f_2 \tag{8}$$

$$w^{(1)}(r,t) = u^{(1)}(r, m_2, t) \tag{9}$$

$$w^{(2)}(r,t) = u^{(1)}(r, m_2, t) \tag{10}$$

$$w^{(1)}(r,t) = u^{(2)}(r, m_1, t) \tag{11}$$

$$w^{(2)}(r,t) = u^{(2)}(r, m_1, t) \tag{12}$$

$$w^{(i)}(r,0) = \varphi_i, \quad i = 1,2 \tag{13}$$

$$w^{(1)}[R(t),t] = w^{(2)}[R(t),t] = \Phi[R(t),t] \tag{14}$$

$$\lambda_1^* \frac{\partial w^{(1)}}{\partial r} \Big|_{r=R(t)} = \lambda_2^* \frac{\partial w^{(2)}}{\partial r} \Big|_{r=R(t)} \tag{15}$$

$$w^{(1)}(r_0, t) = \Upsilon(t), \tag{16}$$

$$w^{(2)}(l,t) = F_2, \tag{17}$$

where $w^{(i)}$ is the temperature, the index $i = 1$ corresponds to the first zone covering the section of the reservoir from the pumping line to the phase transformation front, and the index $i = 2$ corresponds to the second zone covering the rest of the reservoir, u $u^{(1)}$, $u^{(2)}$ are the temperatures of the roof and bottom of the reservoir, $V_1 = \frac{QC_n}{2\pi r \gamma_i C_i^* H}$, $V_2 = \frac{QC_{ж}}{2\pi H}$ is the characteristic velocity of convective heat transfer in the porous medium. Q - the given mass flow rate of the pumped water vapor, H - reservoir thickness, $\lambda_i^* C_i^*$ - heat conductivity and volumetric heat capacity of the saturated porous medium, respectively;

$c_i \lambda_i$ - heat capacity and heat conductivity of the roof and bottom of the reservoir, $c_n \gamma_1$ - volumetric heat capacity and specific gravity of the vapor, $c_{ж} \gamma_2$ - volumetric heat capacity and specific gravity of the liquid, $\varphi_i \gamma_i$ - known sufficiently smooth functions, $\Phi[R(t),t] = \Phi_k$ - condensation temperature, which depends on pressure, $R(t)$ - phase boundary, determined from the condition [1]:

$$\lambda_1^* \frac{\partial W^{(1)}}{\partial r} \Big|_{r=R(t)} + \{C_r \gamma_1 m_n + \gamma_1 m_n P[R(t),t] \left[\frac{1}{\gamma_1} + \frac{1}{\gamma_2} \right] \frac{dR}{dt} \} = \lambda_2^* \frac{\partial W^{(2)}}{\partial r} \Big|_{r=R(t)} + m_n (V_1 C_n - V_2 C_{ж}) * \tag{18}$$

$$* \Phi[R(t),t] \Big|_{z=R(t)}$$

where: C_r - hidden condensation heat, m_n -porosity, $V_1 = \frac{Q}{2\pi H \gamma_1 R(t)}$ - vapor velocity on $R(t)$

$R(t)$, $V_2 = \dfrac{Q}{2\pi H \gamma_2 R(t)}$ - condensate velocity on $R(t)$, $P[R(t),t]$ - pressure at the boundary of $R(t)$, which is determined according to the formula, $P = P_{эк} + f \ln \dfrac{R_k}{R(t)}$. $f = \dfrac{Q}{\mu 2\pi K H \gamma_2}$, k- oil reservoir permeability, μ - average viscosity for liquid phases, $P_{эк}$ - pressure at the bottom of the production well, R_k - distance between the injection and production wells.

Problem (1) - (18) is a generalization of the problem considered in work [1], where heat losses through the roof and bottom of the reservoir were neglected.

It should be noted that generalized problems of filtration theory, in particular, the Veritin problem, can be reduced to problems of this kind [32].

Thus, the problem of determining the temperature field of the reservoir and the boundary of the vapor-condensate zone reduced to Stefan's two-phase problem.

Methods for solving the problem. Since the posed problem is complex, obtaining a solution in analytical form is difficult. Therefore, before proceeding directly to the solution of the problem (1) − (18), we introduce dimensionless variables:

$$\bar{r} = \frac{r}{R_x},\ \bar{R}(t) = \frac{R(t)}{R_x},\ t = \frac{\lambda_1^* t}{C_1^* R_x^2},\ G_1 = \frac{R_x \lambda_1}{H \lambda_1^*},$$

$$G_2 = \frac{R_x \lambda_2}{H \lambda_1^*},\ G_3 = \frac{R_x \lambda_1 C_1^*}{H C_2^* \lambda_2}, G_4 = \frac{R_x \lambda_2 C_1^*}{H C_2^* \lambda_1^*},\ \bar{z} = \frac{z}{R_x},$$

$$\bar{\tau} = \frac{\tau}{R_x},\ l = \frac{1}{R_x},\ A_1 = \frac{C_1^* \lambda_1}{\lambda_1^* C_1},\ A_2 = \frac{C_1^* \lambda_2}{\lambda_1^* C_2},\ A = \frac{C_1^* \eta_2}{\lambda_1^* C_2},$$

$$V_1 = \frac{QC_n}{2\pi H \gamma_1 \lambda_1^*},\ V_2 = \frac{QC_n C_2^*}{2\pi H \gamma_2 \lambda_2^* C_1^*},\ U^{(i)} = \frac{U^{(i)}}{T_i},$$

$$W^{(i)} = \frac{W^{(i)}}{T_i},\ i = 1,2.\ v = m_n(\gamma_1 - \gamma_2 C),\ (\gamma_1 - \gamma_2 C),$$

$$C = \frac{C_2}{C_1}.$$

In dimensionless quantities, the problem (1) − (18) can be rewritten as follows (in the future, the lines above the dimensionless quantities are omitted):

$$\frac{\partial u^{(1)}}{\partial t} = a_1 \frac{\partial^2 u^{(1)}}{\partial z^2};\ (m'' \le z \le m,\ 0 \le t \le T_{bp});$$

(19)

$$\frac{\partial W^{(1)}}{\partial \tau} = \left(\frac{\partial^2 W^{(1)}}{\partial r^2} + \frac{1 - v_1}{r} \frac{\partial W^{(1)}}{\partial r} + G_1 \frac{\partial u^{(1)}}{\partial z}\bigg|_{z=m''} - \right.$$
$$\left. - G_2 \frac{\partial u^{(2)}}{\partial z}\bigg|_{z=m'} \right);\quad (r_0 < r < R(t),\ 0 < t < T_{bp});\qquad (20)$$

$$\frac{\partial W^{(2)}}{\partial \tau} = \left(\frac{\partial^2 W^{(2)}}{\partial r^2} + \frac{a - v_2}{r} \frac{\partial W^{(2)}}{\partial r} + G_3 \frac{\partial u^{(1)}}{\partial z}\bigg|_{z=m''} - \right.$$
$$\left. - G_4 \frac{\partial u^{(2)}}{\partial z}\bigg|_{z=m'} \right);\quad (R(t) < r < 1,\ 0 < t < T_{bp});\qquad (21)$$

$$\frac{\partial u^{(2)}}{\partial t} = a_2 \frac{\partial^2 u}{\partial z^2};\ (0 < z < m',\ 0 < t < T_{bp});\qquad (22)$$

The initial and boundary conditions are written:

$$u^{(i)}(r,z,0) = f_i,\ W^{(1)}\big|_{t=0} P_i,\quad i = 1,2;$$
$$u^{(1)}(r,m_n,t) = f_1,\ u^{(2)}(r,0,t) = f_2,\ W^{(1)}(r_0,t) = \psi_1,$$
$$W^{(2)}(1,t) = \psi_2,\ W^{(1)}(r,t) = u^{(1)}(r,m'',t),$$
$$W^{(1)}(r,t) = u^{(2)}(r,m',t),\ W^{(2)}(r,t) = u(r,m'',t),$$
$$W^{(2)}(r,t) = u^{(2)}(r,m'',t),\ W^{(1)}[R(t),t] = W^{(2)}[R(t),t].$$

The phase transition boundary shift equation has the following form:

$$\frac{dR}{dt} = \frac{\varphi_1 C_1^*}{m_n\{C_2\gamma_2 + P[R(t)](1 - \frac{\gamma_1}{\gamma_2}\}} \left[\frac{\lambda_1}{\lambda_2} \frac{\partial W^{(2)}}{\partial r}\bigg|_{r=R(t)} - \right.$$
$$\left. - \frac{\partial W^{(1)}}{\partial r}\bigg|_{r=R(t)} + \overline{v} R^{-1}(t) \Phi R(t) \right].\qquad (23)$$

For numerical integration, the task is to introduce new variables as follows:

$$\xi = \frac{r - r_0}{R(t) - r_0},\quad \gamma = \frac{r - R(t)}{1 - R(t)} + 1.$$

and then the problem (19) − (23) can be written as

$$\frac{\partial u^{(1)}}{\partial t} = a_1 \frac{\partial^2 u^{(1)}}{\partial z^2};\ (m'' < z < m,\ 0 < t < T_{bp});\qquad (24)$$

$$\frac{\partial u^{(2)}}{\partial t} = a_2 \frac{\partial^2 u^{(2)}}{\partial z^2};\ (0 < z < m',\ 0 < t < T_{bp}); \quad (25)$$

$$\frac{\partial W^{(1)}(\xi,t)}{\partial t} = a_1 \frac{\partial^2 W^{(1)}(\xi,t)}{\partial \xi^2} + b_2(R,\xi)\frac{\partial W^{(1)}(\xi,t)}{\partial \xi^2} + \\ + \varepsilon_6 \frac{\partial u^{(1)}}{\partial z}\Big|_{z=m''} - \varepsilon_6 \frac{\partial u^{(2)}}{\partial \gamma}\Big|_{z=m};\ (0 < \xi < 1,\ 0 < t < T_{bp}) \quad (26)$$

$$\frac{\partial W^{(2)}(\gamma,t)}{\partial t} = a_2(R)\frac{\partial W(\gamma,t)^{(2)}}{\partial \gamma^2} + b_2(R,\gamma) * \\ * \frac{\partial W^{(2)}(\gamma,t)}{\partial \gamma^2} + \sigma_3 \frac{\partial u^{(1)}(\gamma,t)}{\partial \gamma}\Big|_{z=m''} - \sigma_4 \frac{\partial u(\gamma,t)^{(2)}}{\partial \gamma}\Big|_{z=m''}; \quad (27)$$

$$1 \le \gamma \le 2$$

$$u^{(1)}\Big|_{t=0} = f_i,\ W^{(1)}\Big|_{t=0} = \varphi_i,\ i = 1,2 \quad (28)$$

$$u^{(1)}\Big|_{z=m} = f_1,\ u\Big|_{z=0} = f_2, \quad (29)$$

$$W^{(1)}\Big|_{\xi=0} = \gamma,\ W^{(2)}\Big|_{\xi=1} = \tilde{f}^{(1)}, \quad (30)$$

$$W^{(1)} = u^{(1)}\Big|_{z=m''},\ W^{(1)} = u^{(2)}\Big|_{z=m''}, \quad (31)$$

$$W^{(2)} = u^{(1)}\Big|_{z=m''},\ W^{(2)} = u^{(2)}\Big|_{z=m'}, \quad (32)$$

$$W^{(1)}\Big|_{\xi=0} = W^{(2)}\Big|_{\xi=1}, \quad (33)$$

$$\frac{dR}{dt} = \frac{\varphi_1 C_1^* m_n^{-1}}{C_2\gamma_2 + P[R(t)](1-\frac{\gamma_1}{\gamma_2})}\left[\frac{\lambda^*_1}{\lambda^*_2}\frac{1}{1-R(t)}\frac{\partial W^{(2)}}{\partial \gamma}\Big|_{\gamma=1} - \frac{1}{R(t)-r_0}\frac{\partial W^{(1)}}{\partial \xi}\Big|_{\xi=1} + \nu R^{-1}(t)\varphi R(t)\right]. \quad (34)$$

where

$$a_1(R) = \frac{1}{(R(t)-z_0)^2},\quad b_1(R,\dot{R},\xi) = \frac{\xi \dot{R}(t)}{R(t)-r_0} + \\ + \frac{1-\gamma_1}{\xi(R(t)-r_0)+r_0}\cdot\frac{1}{R(t)-r},$$

$$a(R) = \frac{a}{(1-R(t))^2},\quad b_2(R,\dot{R},\zeta) = \frac{\dot{R}(t)(2-\gamma)}{1-R(t)} + \\ + \frac{a-\nu_2}{(\zeta-1)(1-R(t))+R(t)}\cdot\frac{1}{1-R(t)}.$$

The system of partial differential equations (24) − (27) with boundary conditions (28) − (32) does not allow for precise integration due to nonlinear conditions. Therefore, to solve the given problem, we will use conservative numerical methods that are effectively implemented in the computing system. For this, we introduce a non-uniform grid:

$$\omega = \left\{ \begin{matrix} h_{i1} = \xi_{i_1+1} + \xi_i,\ h_{i_2} = h_{i_2+1} - h_{i_2}, \\ h_j = z_{j+1} - z_j,\ \tau_k = t_{k+1} - tk \end{matrix} \right\}$$

The derivatives included in the equations (25) − (28) are approximated as follows:

$$\frac{\partial u^{(1)}}{\partial z}\Big|_{z=m''} = -\left(\left(h'^2_{n_1+1} + 2h'_{n_1} + h'_{n_1+1}\right)u_{n_1,k} + (h'_{n_1+1} + \right. \\ \left. + h'_{n_1})u_{n_1+1,k} - h^2_{n_1}u_{n_1+1,k}\right)/h'_{n_1+1}h'_{n_1}\left(h'_{n_1+1} + h'_{n_1}\right); \quad (35)$$

$$\frac{\partial u^{(2)}}{\partial z}\Big|_{z=m'} = -\left(\left(h_{n_1-1} + h_{n_1-2}\right)^2 \bar{u}_{n_1-1,k} + h^{/2}_{n_1}\bar{u}_{n_1-2,k} + \right. \\ \left. + \left(h'^2_{n_1-2} + 2h_{n_1-1}h_{n_1-2}\right)\bar{u}_{n,k}\right)/h_n h_{n-2}\left(h_{n-1} + h_{n-2}\right); \quad (36)$$

$$u_{j,k} = u^{(1)}\left(\xi_i, z_j, t_k\right),\quad \bar{u}_{j,k} = u^{(2)}\left(\xi_i, z_j, t_k\right).$$

Finite-difference approximation of equations (24) - (27), (34):

$$a_j^{(1)}u_{j-1,k+1} - c_j^{(1)}u_{j,k+1} + b_j^{(1)}u_{j+1,k+1} = -f_j^{(1)}; \quad (37)$$

$$a_j^{(2)}\vec{u}_{j-1,k+1} - c_j^{(2)}\bar{u}_{j,k+1} + b_j^{(2)}\bar{u}_{j+1,k+1} = -f_j^{(2)}; \quad (38)$$

where:

$$a_j^{(i)} = \frac{a_i\tau_k}{h_j h'_{j-1}},\quad c_j^{(i)} = \frac{a_i\tau_k}{h_j^2} + \frac{a_i\tau_k}{h_j h'_{j-1}} + 1,$$

$$a_j^{(i)} = \frac{a_i\tau_k}{h'_j h'_{j+1}},\quad f_j = 0.5\left(h'_j + h'_{j-1}\right),\quad i = 1,2.$$

Equation (37) and (38) taking into account the boundary conditions has the following form:

$$u_{j+1} = d_j^{(1)}u_j + f_j^{(1)},\quad j = 0,n. \quad (39)$$

$$\bar{u}_j = d_{j+1}^{(2)}\bar{u}_{j+1} + e_{j+1}^{(2)},\quad j = 0,n. \quad (40)$$

where:

$$d_{j+1}^{(i)} = \frac{b_j^{(i)}}{c_j^{(i)} - a_j^{(i)}d_j},\ e_{j+1}^{(i)} = \frac{a_j^{(i)}e_j^{(i)} + f_j^{(i)}}{c_j^{(i)} - a_j^{(i)}d_j^{(i)}}.$$

From the relations (40) we can sequentially determine

$$d_{n-1}^{(1)}, d_{n-2}^{(1)}, \ldots, d_n^{(1)};\quad e_{n-1}^{(1)}, e_{n-2}^{(1)}, \ldots, e_n^{(1)};$$

$$d_0^{(2)}, d_1^{(2)}, \ldots, d_{N_1}^{(2)}; \quad e_0^{(2)}, e_1^{(2)}, \ldots, e_m^{(2)}.$$

Introducing into (35) and (36) instead of and their values from (40) and (41), as well as taking into account conditions (28) we introduce:

$$\frac{\partial u^{(1)}}{\partial z}\Big|_{z=m''} = \beta_1 W - \varphi_1^*; \tag{41}$$

$$\frac{\partial u^{(2)}}{\partial z}\Big|_{z=m'} = \beta_2 \bar{W} - \varphi_2^*. \tag{42}$$

Here:

$$\beta_1 = \frac{1}{\mu_1}\left[-\left(h_{n_1}'^2 + 2h_{n_1}' h_{n_1+1}'\right) + \left(h_{n_1+1}' + h_{n_1}'\right)^2 d_{n_1}^{(1)} - \right.$$
$$\left. - h_{n_1}'^2 d_{n_1+1}^{(1)} + d_{n_1}^{(1)} d_{n_1-1}^{(1)}\right];$$

$$\beta_2 = \frac{1}{\mu_2}\left[-\left(h_{n_1-1}'^2 + h_{n_1-2}'\right) d_{n_1}^2 - (h_{n_1-2}'^2 + \right.$$
$$\left. + 2h_{n_1-1}' h_{n_1 21}') - h_{n_1}'^2 d_{n_1-1}^{(2)} d_{n_1}^{(2)}\right];$$

$$\varphi_1^* = \frac{1}{\mu_2}\left[h_{n_1-1}'^2 E_{n_1+1}^{(1)} - h_{n_1}'^2 D_{n_1+1}^{(1)} E_{n_1}^{(1)} - \right.$$
$$\left. - \left(h_{n_1+1}' + h_{n_1}'\right) E_{n_1}^{(1)}\right];$$

$$\varphi_2^* = \frac{1}{\mu_1}\left[h_{n_1-1}'^2 E_{n_1+1}^{(2)} D_{n_1+1}^{(2)} - \left(h_{n_1+1}' + h_{n_1-2}'\right)^2 E_{n_1}^{(2)}\right];$$

$$\mu_1 = \left(h_{n_1}' + h_{n_1+1}'\right)^2 h_{n_1}' - h_{n_1}^3 - h_{n_1+1}' h_{n_1}^2;$$

$$\mu_2 = \left(h_{n_1-1}' - h_{n_1-2}'\right) h_{n_1-1}' h_{n_1-2}'.$$

The finite-difference approximation of equations (34) taking into account (41) and (42) in the non-uniform grid has the form:

$$\mathrm{a}_i^{(S)} \mathrm{W}_{i-1}^{(S+1)} - \tilde{c}_i^{(S)} \mathrm{W}_i^{(S+1)} + \tilde{b}_i^{(S)} \mathrm{W}_{i+1}^{(S+1)} = -f_i; \tag{43}$$

$$\tilde{\mathrm{a}}_i^{(S)} \overline{\mathrm{W}}_{i-1}^{(S+1)} - \tilde{c}_i^{(S)} \overline{\mathrm{W}}_i^{(S+1)} + \tilde{b}_i^{(S)} \overline{\mathrm{W}}_{i+1}^{(S+1)} = -f_i; \tag{44}$$

where:

$$W_{i,\kappa} = W^{(1)}(\xi_i, t_\kappa), \quad \bar{W}_{i,k} = \mathrm{W}^{(2)}(\eta_i, \mathrm{t}_\kappa),$$

$$\tilde{a}_i^{(S)} = \frac{\tau_k}{h_i\left(R_{k+1}^{(S+1)} - r_0\right)}\left\{\frac{1}{h_i\left(R_k^{(S+1)} - r_0\right)} - \right.$$
$$\left. - \left[\frac{\xi_i\left(R_k^{(S+1)} - R_k\right)}{\tau_k} + \frac{1-2\upsilon_1}{\xi_i\left(R_k^{(S+1)} - r_0\right)}\right]\right\}$$

$$\tilde{b}_i^{(S)} = \frac{\tau_k}{h_i\left(R_{k+1}^{(S)} - r_0\right)}\left(\frac{1}{h_i\left(R_{k+1}^{(S)} - r_0\right)} + \right.$$
$$\left. + \frac{\xi_i\left(R_{k+1}^{(S)} - R_k\right)}{\tau_k} + \frac{1-2\upsilon_1}{\xi_i\left(R_{k+1}^{(S)} - r_0\right) + r_0}\right);$$

$$\tilde{c}_i^{(S)} = 1 + \frac{\tau_k}{h_i\left(R_{k+1}^{(S)} - r_0\right)^2}\left(\frac{1}{h_{i+1}} + \frac{1}{h_i}\right) - \frac{\tau_k}{2\left(R_{k+1}^{(S)} - r_0\right)} \times$$
$$\times\left[\frac{\xi_i\left(R_{k+1}^{(S1)} - R_k\right)}{\tau_k} + \frac{1-2\gamma_1}{\xi_i\left(R_{k+1}^{(S)} - r_0\right) + r_0}\right]\left(\frac{1}{h_i} - \frac{1}{h_{i+1}}\right) +$$
$$+\left(\sigma_2\beta_2 - \sigma_j\beta_1\right)\tau_k;$$

$$a_i^{(S)} = \frac{\tau_k}{h_i\left(l - R_{k+1}^{(S)}\right)^2 h_i - 2h_i\left(l - R_k^{(S)}\right)}\left[\frac{(2-h_i)}{\tau_k} * \right.$$
$$\left. * \left(R_{k+1}^{(S)} - R_k\right) + \frac{a-2\upsilon_2}{\left(\eta_i - 1\right)\left(e - R_{k+1}^{(S)}\right) + R_{k+1}^{(S)}}\right]\tau_k;$$

$$\tilde{\tilde{b}}_i^{(S)} = \frac{a\tau_k}{h_i\left(l - R_{k+1}^{(S)}\right) + h_{i+1}} + \frac{\tau_k}{2h_i h_{i+1}\left(l - R_k^{(S)}\right)} *$$
$$* \left(R_{k+1}^{(S)} - R_k\right) + \frac{a-2\upsilon_2}{\left(\eta_i - 1\right)\left(l - R_{k+1}^{(S)}\right) + R_{k+1}^{(S)}};$$

$$\tilde{\tilde{c}}_i^{(S)} = 1 + \frac{a\tau_k}{h_i\left(l - R_{k+1}^{(S)}\right)^2}\left(\frac{1}{h_{i+1}} + \frac{1}{h_i}\right) - \frac{\tau_k}{2\left(l - R_{k+1}^{(S)}\right)} \times$$
$$\times\left[\frac{R_{k+1}^{(S)} - R_k}{\tau_k}(2-h_i) + \frac{a-2\upsilon_2}{\left(\eta_i - 1\right)\left(l - R_{k+1}^{(S)}\right) + R_{k+1}^{(S)}}\right] *$$
$$*\left(\frac{1}{h_i} - \frac{1}{h_{i+1}}\right) + \left(\beta_2\sigma_4 - \beta_1\sigma_3\right)\tau_k;$$

$$f_i = W_i + \sigma_2\varphi_2^*\tau_k - \varphi_1^*\sigma_1\tau_k;$$
$$f_i = \overline{W_i} + \sigma_4\varphi_2^*\tau_k - \varphi_1^*\sigma_3\tau_k.$$

To find the unknown boundary, we have the equation

$$R_{x+1}^{(S+1)} = \tau_k\left(g_{x+1}^{-1}\right)^{(S)}\left[\lambda^*_{i+1} h_{i+1}^{-1}\left(l - R_{x+1}^{(S)}\right)^{-1}\left(W_{n^*}^{(S+1)} - \right.\right.$$
$$\left. - 4W_{n^*+1}^{(S+1)} + W_{n^*+2}^{(S+1)}\right) - h^{-1}{}_i\left(R_{x+1}^{(S)} - r_0\right)^{-1}\left(R_{x+1}^{(S)} - r_0\right)^{-1} *$$
$$\left.\left(4W_{n^*-1}^{(S+1)} - 3W_{n^*}^{(S+1)} - W_{n^*-2}^{(S+1)}\right) + \upsilon\left(R_{x+1}^{-1}\right)^S \Phi_{x+1}^{(S+1)} + R_k\right];$$

where

$$g_{x+1}^{(S)} = \frac{m_n}{\varphi_1 c_1^*}\left[c_r\gamma_1 + \left(P_{ipr} + f\ln\frac{1}{R_{k+1}^{(S)}}\right)\left(1-\frac{\gamma_1}{\gamma_2}\right)\right].$$

Note that the scheme (37) − (38) on an equivalent grid has a second order of accuracy for spatial variables. From (43) and (44), it is clearly evident that the coefficients of these equations satisfy the conditions for the stability of the proponent formulas, i.e.

$$a_j^{(i)} > 0, \quad b_j^{(i)} > 0, \quad c_j^{(i)} > a_j^{(i)} + b_j^{(i)},\ i = 1,2. \qquad (45)$$

From these inequalities, it can be concluded that $D_j^{(i)} \le 1$.

From this follows $\beta_1 \ge 0, \quad \beta_2 \ge 0$.

Now it is not difficult to notice that the coefficients of the system of equations (43) and (44) satisfy the stability conditions of the distillation formulas (44) with a monotonic nature of the boundary change. Therefore, we will solve the system of equations (43) and (44) by the iterative distillation method:

$$W_i^{(S+1)} = d_{i+1}^{(S)} W_{i+1}^{(S)} + \tilde{e}_{i+1}^{(S)}; \qquad (46)$$

$$\overline{W}_i^{(S+1)} = d_{i+1}^{(S)} \overline{W}_{i+1}^{(S)} + \tilde{\tilde{e}}_{i+1}^{(S)}; \qquad (47)$$

where

$$d_{i+1}^{(S)} = \frac{\tilde{b}_i^{(S)}}{\tilde{c}_i^{(S)} - \tilde{a}_i^{(S)} d_i^{(S)}};\ \tilde{e}_{i+1}^{(S)} = \frac{\tilde{a}_i^{(S)} \tilde{e}_i^{(S)} + \tilde{F}_i^{(S)}}{\tilde{c}_i^{(S)} - \tilde{a}_i^{(S)} d_i^{(S)}}; \qquad (48)$$

$$d_{i+1}^{(S)} = \frac{\tilde{\tilde{b}}_i^{(S)}}{\tilde{\tilde{c}}_i^{(S)} - a_i^{(S)} d_i^{(S)}};\ \tilde{\tilde{e}}_{i+1}^{(S)} = \frac{a_i^{(S)} \tilde{\tilde{e}}_i^{(S)} + F_i^{(S)}}{\tilde{\tilde{c}}_i^{(S)} - a_i^{(S)} d_i^{(S)}}. \qquad (49)$$

Then, using formulas (46) and (47), after calculating $d_i^{(S)}$ and $e_i^{(S)}$ using recurrent relations (48) and (49), the value of $W_1^{(S+1)}, W_2^{(S+1)}, \ldots, W_{n_1}^{(S+1)}, W_1^{(S+1)}, W_n^{(S+1)}$ is sequentially determined.

The iteration process will end when the condition is met

$$\max\left\{\left|W_i^{(S+1)} - W_i^{(S)}\right|, \left|\tilde{W}_i^{(S+1)} - \tilde{W}_i^{(S)}\right|\right\} < \varepsilon.$$

All values of the function u_j and $\bar{u}_j$ are calculated using formulas (40) and (41).

Analysis of numerical calculations according to the algorithm described above shows the feasibility of applying damping by formulas during iterations.

$$W_i^{(S+1)} = \mu^* \tilde{W}_i^{(S+1)} + \left(1-\mu^*\right) W_i^{(S)};$$

$$\tilde{W}_i^{(S+1)} = \mu^* \overline{\tilde{W}}_i^{(S+1)} + \left(1-\mu^*\right) \tilde{W}_i^{(S)};$$

where - $W_i^{(S+1)}$ and $\tilde{W}_i^{(S+1)}$ are the solutions obtained from (46) and (47).

When assessing the accuracy of the numerical solution, the balance relations method is used.

To derive this relationship, let's proceed as follows. Integrating equation (24) with respect to ξ in the interval (0,1) taking into account the boundary conditions (25) and (27), we obtain:

$$\frac{dW_{cp}}{dt} = a_1(R)\Phi(t) - a_1(R)Y^* + \int_0^1 B_1\left(R,\dot{R},\xi\right)\frac{dw}{d\xi}d\xi + q_1(t) - q_2(t) \qquad (50)$$

where

$$q_1 = \sigma_1 \frac{\partial u_{cp}^{(1)}}{\partial z}\Big|_{z=m''};\ q_2 = \delta_2 \frac{\partial u_{cp}^{(2)}}{\partial z}\Big|_{z=m'}; \qquad (51)$$

$$u_{cp}^{(i)} = \int u^{(i)} d\xi; \quad W_{cp} = \int_0^1 w d\xi;\ i = 1,2,.. \qquad (52)$$

Integrating (51) for *t*, we get:

$$W_{cp} = \int_0^t \left[a_1(R)\Phi(t) - a_1(R)Y^* + q_1(\tau) - q_2(\tau)\right]d\tau + \int_0^t\int_0^1 b_1\left(R,\dot{R},\xi\right)\frac{\partial W}{\partial \xi}d\xi d\tau.$$

To evaluate the accuracy of the numerical solution by this equation, we proceed as follows. The numerical solution corresponding to the time moment *t*, averaged by formulas (51), is then calculated using (52), and by substituting these quantities into (50) and calculating the double integral, we obviously obtain an approximate equality characterizing the accuracy of the approximate solution.

Numerical experiments show that the (37) - (43) scheme is monotonic even at small flow rates when $Q < 1000$ kg/hour.

Next, we construct another second-order approximation scheme that is monotonic for any *Q*.

For this equation (26) and (27), we represent it as a system:

$$\begin{cases} \dfrac{\partial V}{\partial t} = a_1 \dfrac{\partial^2 V}{\partial \xi^2} + B\dfrac{\partial V}{\partial \xi} + G^{(1)}; \\ \dfrac{\partial \tilde{V}}{\partial t} = a_2 \dfrac{\partial^2 \tilde{V}}{\partial \xi^2} + B\dfrac{\partial \tilde{V}}{\partial \xi} + G^{(2)}; \end{cases} \quad (53)$$

where

$$V = \begin{pmatrix} v^{(1)} \\ v^{(2)} \end{pmatrix}; \quad \tilde{V} = \begin{pmatrix} \tilde{v}^{(2)} \\ \tilde{v}^{(2)} \end{pmatrix}; \quad B = \begin{pmatrix} 0 & B_1 \\ B_1 & 0 \end{pmatrix};$$

$$\tilde{B} = \begin{pmatrix} 0 & B_2 \\ B_2 & 0 \end{pmatrix};$$

$$G^{(i)} = \begin{pmatrix} g^{(1)} \\ g^{(2)} \end{pmatrix}; \quad g^{(i)} = \sigma_{2i} \frac{\partial u^{(1)}}{\partial z}\Big|_{z=m''} - \sigma_{2i} \frac{\partial u^{(2)}}{\partial z}\Big|_{z=m'};$$

$i = 1, 2.$

The boundary conditions in this case will take the form:

$$V_{\xi=0} = Y; \quad \tilde{V}_{\eta=2} = f_1; \quad V_{t=0} = \varphi_1;$$

$$\tilde{V}_{t=0} = \varphi_2; \; V\big|_{\xi=1} = \tilde{V}_{\eta=1}.$$

It is not difficult to notice that

$V^{(1)} = V^{(2)} = W^{(1)}$ and $V^{(1)} = V^{(2)} = W^{(2)}$

The derivatives included in the system (54) are replaced by difference relations. Then we obtain systems of finite-difference equations

$$\begin{cases} \dfrac{v_{k+1}^{(1)} - v_k^{(1)}}{\tau} = A_1 \Lambda v_{k+1}^{(1)} + B_1 \Lambda v_{k+1}^{(2)} + g_{k+1}^{(1)}; \\ \dfrac{v_{k+1}^{(2)} - v_k^{(2)}}{\tau} = A_1 \Lambda v_{k+1}^{(1)} + B_1 \Lambda v_{k+1}^{(2)} + g_{k+1}^{(2)}; \\ \dfrac{\tilde{v}_{k+1}^{(1)} - \tilde{v}_k^{(1)}}{\tau} = A_2 \Lambda \tilde{v}_{k+1}^{(1)} + B_2 \Lambda \tilde{v}_{k+1}^{(2)} + g_{k+1}^{(3)}; \\ \dfrac{\tilde{v}_{k+1}^{(2)} - \tilde{v}_k^{(2)}}{\tau} = A_2 \Lambda \tilde{v}_{k+1}^{(2)} + B_2 \Lambda \tilde{v}_{k+1}^{(1)} + g_{k+1}^{(4)}; \end{cases} \quad (54)$$

Here

$$\Lambda_1 v_{k+1}^{(2)} = \frac{v_{k+1,i+1}^{(2)} - v_{k+1,i}^{(2)}}{h}; \quad \Lambda_2 \tilde{v}_{k+1}^{(2)} = \frac{\tilde{v}_{k+1,i}^{(2)} - \tilde{v}_{k+1,i-1}^{(2)}}{h};$$

$$g_{k+1}^{(1)} = (\sigma_1\beta_1 - \sigma_2\beta_2) v_{k+1}^{(1)} - \varphi_1^* \sigma_1 + \varphi_2^* \sigma_2;$$

$$g_{k+1}^{(2)} = (\sigma_1\beta_1 - \sigma_2\beta_2) v_{k+1}^{(1)} - \varphi_1^* \sigma_1 + \varphi_2^* \sigma_2;$$

$$g_{k+1}^{(3)} = (\sigma_3\beta_1 - \beta_2\sigma_4) v_{k+1}^{(1)} - \varphi_1^* \sigma_3 + \varphi_2^* \sigma_4.$$

For solving the system of equations (54), the method described above was used. Numerical experiments show that the scheme (54) is stable for any values of $Q \geq 0.$

Results and analysis.

As is known, the process of heat propagation in the reservoir when water vapor is pumped into it is somewhat more complex than when hot water is pumped. Therefore, the steam injection process has been studied to a significantly lesser extent than the hot water injection process.

The works [1, 2, 3, 4, 6] and others are devoted to the experimental and theoretical study of the heat distribution process in the reservoir during water vapor injection. As shown in these works, when steam is pumped from the reservoir, 3-4 times more oil can be extracted than when watered.

Mathematical description of the heat propagation process in the reservoir when steam is supplied to it leads to the solution of complex Stefan-type problems, the difficulty of solving which is explained by the fact that the law of movement of the phase transition boundary is not known in advance and is sought.

In the work [1], the problem (1) - (16) was solved numerically for the case of a "thermal insulated" layer, i.e., it was assumed that the thermal conductivity of the roof and bottom of the layer is equal to zero. However, it should be noted that such an assumption is acceptable only for the case of short-term steam injection, for example, during steam-thermal treatment of the bottomhole zone of oil wells.

To assess the influence of heat loss accounting through the roof and bottom of the layer, the task was considered more precisely: the thermal conductivity of the covering and underlying rocks was considered in the vertical direction.

Figure 2 shows the temperature distribution curves in the reservoir when water vapor is pumped into it. The calculations were performed for the following parameter values:

$\lambda_1^* = 0.1$ kcal/hour $\cdot$°C, $\lambda_2^* = 1.5$ kcal/hour $\cdot$°C,

$c_1^* = 1.5$ kcal/m$^3 \cdot$°C, $c_2^* = 500$ kcal/m$^3 \cdot$°C,

$\mu^* = 22.5sPz.\ 28sPz.\ 45sPz.,\ K = 0.1\ Darsi,$

$\gamma_1^* = 5$ kg/m^3, $\gamma_2^* = 1000$ kg/m^3, $c_H^* = 4$ kcal/m$^3 \cdot$°C,

$c_H = 1400$ kcal/m$^3 \cdot$°C, $\lambda_1 = 0.86$ kcal/m $\cdot$ *hour* $\cdot$°C,

$\lambda_2 = 0.86$ kcal/m $\cdot$ *hour* $\cdot$°C, $c_2 = 500$ kcal/m$^3 \cdot$°C,

$C_z = 500$ kcal/m$^3 \cdot$°C, $T_0 = 50$°C, $C_r = 480$ kcal/kg,

$P_{эк} = 50atm.,\ R_k = 100m,\ Q = 2000kg/h,\ 3000kg/h.$

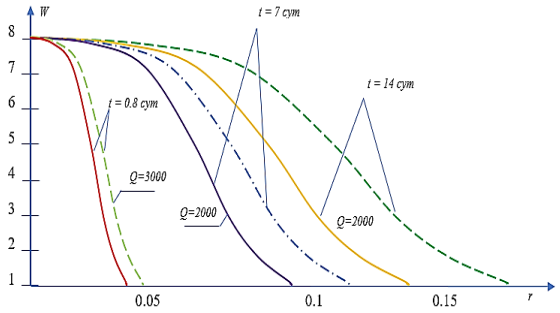


Fig. 2. Temperature distribution curves in the reservoir.

The calculation results show that the nature of temperature distribution in the reservoir strongly depends on the steam flow rate. With its growth, the heating depth naturally increases.

The role of heat loss accounting through the roof and bottom of the reservoir can be traced by comparing the temperature distribution curves in the reservoir constructed from the solution of problem (24) - (34) with the curves obtained in the work [1]. This comparison is presented in Figure 2.

Here, the curves from the calculations of problem (24) - (34) are constructed using dotted lines, and the solid lines are constructed using the calculated data obtained in the work [1].

From Figure 3, it can be seen that accounting for heat loss through the roof and bottom of the reservoir leads to a decrease in the calculated temperature, and with the increase in the thermal zone, heat loss increases and the rate of heat loss increases. Furthermore, unlike the curves obtained in [1], in our case, the sharp declines disappear, and the curves become more monotonous.

Calculations, taking into account heat losses through the roof and bottom of the reservoir, yield results significantly different from those obtained in the work [1]. The maximum absolute error at variation t reaches 0.1 at t = 7 days. With increasing time, this error increases. As can be seen from Figure 3, large discrepancies in heat loss accounting are especially noticeable in the liquid zone. Obviously, this discrepancy is explained by the insufficient description of temperature distribution in the problem statement [1], especially in the case of a prolonged heat carrier injection process. Note that the temperature in the steam zone, when heat losses through the roof and bottom of the reservoir are neglected, remains practically constant, equal to the temperature of the steam being pumped. When accounting for heat losses, the temperature of the vapor zone changes significantly over time.

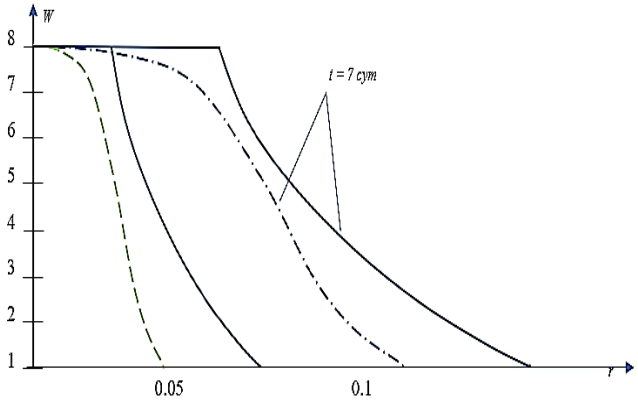


Fig. 3 Temperature distribution curves in the reservoir and their compilation using the method given in the work [1]

– · –calculated curve; ——— method curve (1).

Figure 4 compares the graphs of the R (t) function, taking into account heat loss through the roof and bottom of the reservoir and without it. Note that the boundary of the regions also depends on the viscosity of the liquid, and with a decrease in viscosity, the vapor zone increases.

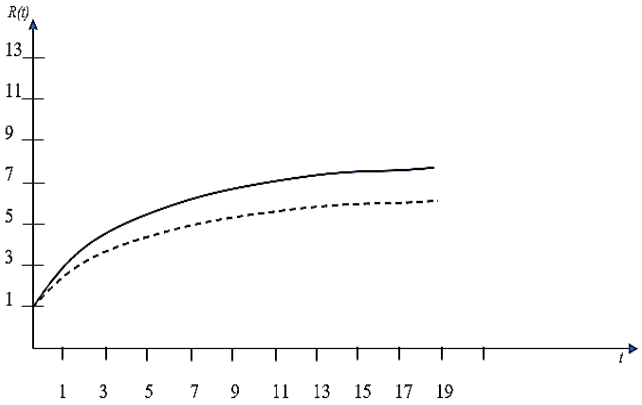


Fig. 4. Change in phase transition boundary shift over time

– – –calculated curve; ——— method curve (2).

CONCLUSION

Numerical calculations have established that accounting for heat loss through the roof and bottom of the reservoir leads to a decrease in the calculated temperature. As observed, as the thermal zone increases, thermal losses increase, and the rate of thermal loss increases over time. Moreover, computational experiments conducted on the computational cluster showed that taking into account heat losses through the roof and bottom of the reservoir leads to results significantly different from those obtained in the work [1].

Analysis of the results showed that large discrepancies in the accounting of heat losses are especially noticeable in the liquid zone and are explained by the insufficient description of temperature distribution in the problem statement in the work [1], especially in the long-term process of heat carrier injection. Calculations have shown that the temperature in the steam zone, when heat losses through the roof and bottom of the reservoir are neglected, remains practically constant, equal to the temperature of the pumped steam, and when heat losses are taken into account, the temperature of the steam zone changes noticeably over time.

It has been established that the boundary of the regions also depends on the viscosity of the liquid, and with a decrease in viscosity, the vapor zone increases.

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